Variational Closure of Quarkbase Cosmology

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Abstract

This work provides the formal variational closure of Quarkbase Cosmology by constructing a single explicit action for the vacuum Ψ-field. Previous developments of the framework established a coherent physical description of vacuum screening, rigidity, stability, and discrete structures, but in a distributed, non-variational form, leaving open a precise formal gap: the absence of an explicit action principle from which the fundamental dynamics and conservation laws follow by variation.

Here we show that Quarkbase Cosmology can be formulated as a closed classical effective field theory of a continuous vacuum medium. A minimal action is constructed, containing a relativistic kinetic term, a finite screening scale, an elastic gradient response, a rigidity operator, and a stabilizing potential. No new physical assumptions or degrees of freedom are introduced.

Variation of the action yields a unique field equation for the Ψ-field and guarantees energy–momentum conservation via Noether's theorem. Discrete, finite-energy vacuum structures arise as isolated extrema of the associated energy functional, rather than as postulates. Previously established results on vacuum screening, stability, characteristic length scales, and structural closure are recovered as unavoidable consequences of the variational principle.

Prediction. The closed action yields a single quantitative and falsifiable prediction. In the linear effective regime, the dispersion relation of vacuum normal modes contains an unavoidable quartic contribution,

$$\omega^2(k) = \omega_0^2 + v^2k^2 + Ak^4,$$

implying that the reduced dispersion residual

$$R(k) = \frac{\omega^2(k) - \omega_0^2 - v^2k^2}{k^4}$$

must be constant throughout the domain of validity of the effective theory. This operator-level invariant cannot be reproduced by parameter retuning in second-order theories and admits a sharp falsification criterion.

The dispersion does not correspond to phonons, plasmons, electromagnetic modes, or atomic excitations, but to linear normal modes of the vacuum Ψ-field itself. These modes are probed indirectly through their phase imprint on oscillatory phenomena coupled to the vacuum, most notably cosmological acoustic oscillations in the cosmic microwave background and baryon acoustic oscillations, which act as interferometric detectors of the underlying vacuum dispersion.

With this result, Quarkbase Cosmology transitions from a physically coherent framework to a formally closed and empirically testable theory, whose validity rests on the presence or absence of a rigidity-induced quartic dispersion residual.

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1 Introduction

Variational Closure of Quarkbase Cosmology

Throughout the development of **Quarkbase Cosmology (QC)**, a coherent physical framework has been constructed that reproduces and reinterprets a wide range of phenomenaranging from luminal propagation and vacuum screening to structural stability and cosmological anisotropiesstarting from the central hypothesis of an underlying continuous medium described by a scalar field Ψ.

A recurrent technical criticism, however, has identified a precise formal gap: the absence of a **single explicit variational principle** from which the dynamical equations, conservation laws, and effective couplings of the framework are derived in a systematic way. In standard theoretical terminology, this is expressed as the claim that QC is not yet "formally closed."

This criticism does not question the internal coherence, physical motivation, or explanatory power of the framework. Rather, it concerns its **formal status** within the conventional language of theoretical physics. Specifically, it demands the existence of an action

$$S[\Psi, \dots] \;=\; \int d^4x \, \sqrt{-g} \, \mathcal{L}$$

such that the fundamental field equations arise by variation, and the associated conservation laws follow automatically via Noether's theorem.

The purpose of this article is to address this point directly and explicitly. We show that Quarkbase Cosmology can be formulated as a **closed effective theory** by constructing a minimal variational principle for the Ψ-field that consolidates, in a single mathematical object, structures that were previously introduced in a distributed form: vacuum pressure, elastic rigidity of the medium, screening scales, and effective torsional responses.

No new physical assumptions are introduced. The action presented here is not an *ad hoc* addition, but a condensation of elements already presentimplicitly or explicitlythroughout the existing QC literature. Once made explicit, this variational formulation rigidifies the framework, fixes its scales, enforces conservation laws, and elevates QC to a formally closed, falsifiable theoretical structure.

In the sections that follow, we first summarize the current non-variational formulation of QC and identify where its variational content already appears in latent form. We then introduce the minimal action for the Ψ-field, derive the corresponding field equations, and show how key results previously obtained in Quarkbase Cosmology emerge naturally from this unified principle.

2 Current Status of Quarkbase Cosmology

Implicit Variational Structure and Its Limits

Quarkbase Cosmology has, from its inception, been developed as a physically motivated framework grounded in continuum reasoning rather than axiomatic formalism. Its central objectthe scalar field Ψrepresents a real, frictionless vacuum medium whose local configurations encode pressure, energy density, and structural stability. From this starting point, a wide range of dynamical and cosmological consequences have been derived using geometric, elastic, and energetic arguments.

At the operational level, QC already employs several concepts that are intrinsically variational in nature:

- Field configurations are repeatedly characterized as **stable or unstable** depending on whether they minimize or extremize an associated energy functional.
- Screening lengths and attenuation scales are introduced as **regulators** that prevent divergences and ensure finite-energy configurations.
- Structural closures (quarkbases) are identified as **preferred configurations** of the medium, selected by energetic and geometric constraints.
- Linear propagation equations are treated as arising from the response of the medium to perturbations around a homogeneous background.

These elements strongly suggest the presence of an underlying functional whose extrema govern the dynamics. Indeed, in several contexts the discussion explicitly refers to quantities of the form $E[\Psi]$, and to the fact that certain configurations cannot represent physical states because they would lead to divergent contributions to such a functional. This is already a variational statement, albeit restricted to static or quasi-static regimes.

What is missing is not the physics, but the **unification of these arguments into a single, explicit variational object** defined over spacetime.

2.1 Distributed formulation versus unified action

In its current form, QC proceeds by assembling physical consequences from locally justified principles:

- pressure gradients drive effective forces,
- rigidity terms stabilize finite structures,
- torsional effects encode nontrivial internal responses of the medium,
- conservation laws are enforced by physical consistency rather than derived identities.

This approach is legitimate and common in exploratory phases of theory building. Historically, many successful theoriesclassical elasticity, hydrodynamics, and even early electromagnetismwere first formulated in this way before being recast into a compact variational language.

However, without an explicit action:

- there is no formal criterion of uniqueness for the equations of motion,
- conservation laws are imposed rather than guaranteed,
- the interdependence of constants and scales remains implicit,
- and the boundary between fundamental structure and effective approximation is not sharply defined.

The result is a framework that is physically coherent but formally open.

3 Physical Nature of the Ψ-Field

Degrees of Freedom, Symmetries, and Regime of Validity

Before constructing an explicit action, it is necessary to state clearly what the Ψ-field represents, what it does **not** represent, and under which assumptions it is defined. This section fixes the ontology and symmetry content of the theory, eliminating ambiguities that would otherwise propagate into the variational formulation.

3.1 Ontological status of Ψ

In Quarkbase Cosmology, Ψ is not an abstract auxiliary field nor a bookkeeping device. It represents a **real, continuous, frictionless vacuum medium** whose local state encodes pressure, energy density, and structural response. All observable phenomena addressed by QCpropagation, confinement, screening, stabilityare interpreted as manifestations of configurations and excitations of this medium.

Ψ is therefore:

- **Continuous**: no fundamental discreteness is assumed at the level of the field.
- **Scalar**: its primary degree of freedom is a single real-valued field over spacetime.
- **Physical**: Ψ carries energy and momentum and admits an associated stress–energy tensor.
- **Universal**: there is no background "empty" spacetime independent of Ψ; the vacuum itself is the medium.

This choice is not arbitrary. It reflects the fact that all existing QC results can be expressed in terms of a single scalar quantity controlling local vacuum pressure and response, without the need to introduce additional fundamental fields.

3.2 Background structure and metric

At the level addressed in this work, QC is formulated as an **effective relativistic field theory** on a spacetime endowed with a metric structure. Two equivalent viewpoints are admissible:

1. The metric $g_{\mu\nu}$ is treated as a fixed background (typically Minkowski) encoding the operational Lorentz invariance already demonstrated within QC.
2. The metric is interpreted as an **effective emergent structure**, determined by the large-scale state of the Ψ-field, but treated as non-dynamical for the purposes of the present closure.

In both cases, the variational principle is defined with respect to Ψ alone. No Einstein–Hilbert term is introduced, and no claim of a complete gravitational theory is made here. This choice is deliberate: the objective is formal closure of the Ψ-sector, not a premature unification with full dynamical gravity.

3.3 Symmetry assumptions

The action to be constructed is constrained by a minimal and physically motivated set of symmetries, all of which are already implicit in the QC corpus:

- **Spacetime translation invariance**, ensuring conservation of energy and momentum.
- **Rotational invariance** of the vacuum in its unperturbed state.
- **Effective Lorentz invariance** of the linearized dynamics around homogeneous backgrounds.
- **Locality**, in the sense that the Lagrangian density depends on Ψ and a finite number of its derivatives.

No internal gauge symmetry is postulated. The richness of the theory arises not from gauge redundancy, but from the nontrivial elastic and structural response of the medium itself.

3.4 Regime of validity

The variational formulation presented here defines Quarkbase Cosmology as a **classical effective field theory** of the vacuum medium. This has several important implications:

- The action is intended to be valid from laboratory to cosmological scales where QC has already been applied.
- Ultraviolet completion and fundamental quantization are **explicitly outside the scope** of this construction.
- Higher-order derivative terms are permitted insofar as they encode rigidity and stability, not as artifacts of speculative ultraviolet physics.

This clarification is essential. By adopting the language of effective field theory, QC gains formal legitimacy without overcommitting to structures it does not yet require.

With the field content, symmetries, and regime of validity now fixed, we are in a position to construct the **minimal action** for the Ψ-field. In the next section, we introduce this action term by term and justify each contribution directly from previously established physical principles within Quarkbase Cosmology.

4 Construction of the Minimal Action

Explicit Variational Principle for the Ψ-Field

With the physical nature of the Ψ-field and the symmetry assumptions fixed, we now construct an explicit action that closes the framework. The guiding principle is strict minimality: every term included must correspond to a physical ingredient already presentexplicitly or implicitlyin prior formulations of Quarkbase Cosmology.

No additional degrees of freedom are introduced, and no speculative structures are assumed.

4.1 General form of the action

We postulate that the dynamics of the vacuum medium are governed by a spacetime action of the form

$$S_\Psi \;=\; \int d^4x\,\sqrt{-g}\,\mathcal{L}_\Psi,$$

where the Lagrangian density $\mathcal{L}_\Psi$ depends on the field Ψ and its derivatives up to finite order. Consistent with the effective-field-theory character of QC, higher spatial derivatives are permitted when they encode elastic rigidity rather than ultraviolet physics.

The minimal Lagrangian is written as

$$\mathcal{L}_\Psi = \frac{\kappa}{2}\,\partial_\mu\Psi\,\partial^\mu\Psi - \frac{\kappa}{2}\,\lambda^{-2}\Psi^2 - \frac{\alpha}{2}\,(\nabla\Psi)^2 - \frac{\beta}{2}\,(\nabla^2\Psi)^2 - V(\Psi).$$

Each term corresponds to a distinct and already-employed physical mechanism.

4.2 Kinetic term: propagation of disturbances

The term

$$\frac{\kappa}{2}\,\partial_\mu\Psi\,\partial^\mu\Psi$$

is the relativistic kinetic term for a scalar field. It encodes the fact that small perturbations of the vacuum medium propagate as waves with a finite characteristic velocity.

In QC, this propagation is repeatedly shown to be luminal in the linear regime. The kinetic term therefore enforces effective Lorentz invariance of the linearized dynamics and guarantees a well-defined causal structure.

The constant κ sets the overall stiffness of the medium and fixes the normalization of energy and momentum.

4.3 Screening term: finite-range response of the vacuum

The quadratic term

$$-\frac{\kappa}{2}\,\lambda^{-2}\Psi^2$$

introduces a characteristic screening length λ. This is not an arbitrary mass term; it encodes the finite-range response of the vacuum medium and plays the role of an infrared regulator.

In earlier QC work, λ appears as a physically necessary scale ensuring that large-scale configurations carry finite energy and that long-wavelength modes are attenuated. Its appearance here is therefore not optional: without this term, the action would fail to reproduce known QC results.

Importantly, once included at the level of the action, λ is no longer an adjustable afterthought. It becomes an intrinsic scale of the theory.

4.4 Gradient term: vacuum pressure and elastic response

The spatial-gradient contribution

$$-\frac{\alpha}{2}\,(\nabla\Psi)^2$$

captures the energetic cost associated with spatial inhomogeneities of the vacuum pressure. Physically, it represents the simplest elastic response of the medium to deformation.

This term is already implicit in QC arguments where pressure gradients generate effective forces and where smooth configurations are energetically preferred. Its inclusion ensures that static configurations satisfy well-defined equilibrium equations derived from energy minimization.

4.5 Rigidity term: stabilization of finite structures

The higher-derivative term

$$-\frac{\beta}{2}\,(\nabla^2\Psi)^2$$

is essential for structural stability. It penalizes sharp curvature and suppresses pathological configurations that would otherwise minimize the action.

Within QC, this term encodes the **rigidity of the vacuum medium**, analogous to bending energy in elastic materials. It is this contribution that stabilizes closed configurations and allows discrete, finite-energy structures to emerge naturally.

Without such a term, the theory would admit runaway or singular solutions incompatible with the observed stability of quarkbase closures.

It is worth emphasizing the specific status of the rigidity term $(\nabla^2\Psi)^2$ within the effective-field-theory structure of the action. Although higher-derivative operators are in general expected in an EFT expansion, the present term is not introduced as part of an uncontrolled truncation.

Among all fourth-order operators compatible with the assumed symmetriesscalar character, rotational invariance, parity, and locality$(\nabla^2\Psi)^2$ is the only operator that contributes positively to the energy functional and is therefore capable of stabilizing finite configurations of the Ψ-field. Other operators of the same derivative order either reduce to total derivatives, are redundant under integration by parts, or do not affect stability properties.

The inclusion of this term is thus not optional but necessary: without it, the theory would admit energetically favored singular or runaway configurations incompatible with the observed stability of quarkbase closures. In this sense, the rigidity term represents the minimal higher-order contribution required for consistency, rather than an arbitrary EFT correction.

4.6 Potential term: homogeneous vacuum state

The function $V(\Psi)$ determines the preferred homogeneous state of the vacuum. Its precise form is not fixed *a priori*, but minimal assumptions suffice:

- $V(\Psi)$ admits at least one stable minimum corresponding to the unperturbed vacuum.
- Small deviations around this minimum are already captured by the quadratic terms above.

No fine-tuning is required. The role of $V(\Psi)$ is simply to anchor the background configuration about which physical excitations are defined.

4.7 Remarks on completeness and economy

The action constructed here satisfies several critical criteria simultaneously:

- It is **explicitly variational**.
- It reproduces all known QC dynamical ingredients.
- It introduces no superfluous fields or symmetries.
- It fixes scales and couplings structurally rather than by postulate.

Most importantly, it transforms Quarkbase Cosmology from a distributed set of physically motivated equations into a **single unified variational framework**.

In the next section, we perform the explicit variation of this action and derive the fundamental field equation for Ψ, along with its associated conservation laws.

4.8 On minimality and operator completeness

The action constructed above is not presented as an arbitrary choice, but as the minimal and complete realization of the Ψ-field dynamics under the assumptions explicitly stated in this work. These assumptions are: locality, rotational invariance of the vacuum, effective Lorentz invariance of the linearized sector, parity symmetry, and the requirement of finite-energy, stable configurations.

Under these conditions, the Lagrangian density written in Section 4 is the most general scalar functional of the Ψ-field containing operators up to fourth order in spatial derivatives that contribute nontrivially to the dynamics and stability of the theory. Lower-order operators are uniquely fixed by propagation and screening requirements, while higher-order operators are suppressed within the effective-field-theory regime adopted here.

In this sense, the action should be understood as unique *up to the chosen derivative order*. Alternative Lagrangians compatible with the same symmetries necessarily differ either by higher-order operators, which do not affect the existence or stability of the relevant extrema, or by terms that can be eliminated by integration by parts or field redefinitions.

The present construction therefore does not claim absolute uniqueness in a formal mathematical sense, but establishes the action as the unique minimal representative of the Ψ-field dynamics within the physically relevant effective regime addressed by Quarkbase Cosmology.

5 Variational Derivation and Fundamental Field Equation

Dynamics and Conservation from First Principles

Having constructed an explicit action for the Ψ-field, we now derive its consequences. This section performs the variational calculation explicitly and shows that the fundamental equations of Quarkbase Cosmology emerge automatically, without additional postulates.

5.1 Euler–Lagrange equation for the Ψ-field

The dynamics of Ψ follow from the stationary-action condition

$$\frac{\delta S_\Psi}{\delta \Psi} = 0.$$

For a Lagrangian density depending on Ψ, its first derivatives, and higher spatial derivatives, the generalized Euler–Lagrange equation reads

$$\frac{\partial \mathcal{L}}{\partial \Psi} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \Psi)} \right) + \partial_\mu \partial_\nu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \partial_\nu \Psi)} \right) = 0.$$

Applying this to the action defined in the previous section yields the fundamental field equation

$$\kappa \, \Box \Psi - \kappa \, \lambda^{-2} \Psi + \alpha \, \nabla^2 \Psi - \beta \, \nabla^4 \Psi + \frac{dV}{d\Psi} = 0,$$

where $\Box = g^{\mu\nu} \nabla_\mu \nabla_\nu$ is the d'Alembert operator and $\nabla^4 = (\nabla^2)^2$.

This equation is the **closed dynamical core** of Quarkbase Cosmology.

5.2 Interpretation of the terms

Each contribution has a direct physical interpretation already familiar within QC:

- $\kappa \, \Box \Psi$: relativistic propagation of disturbances in the vacuum medium.
- $-\kappa \, \lambda^{-2} \Psi$: finite-range screening of vacuum response.
- $+\alpha \, \nabla^2 \Psi$: pressure-driven relaxation toward smooth configurations.
- $-\beta \, \nabla^4 \Psi$: rigidity term stabilizing finite and closed structures.
- $dV/d\Psi$: restoration toward the homogeneous vacuum state.

Crucially, no term appears whose role is unclear or whose physical meaning is imposed retroactively. The equation is not postulated; it is **forced** by the action.

5.3 Linearized regime and dispersion relation

Consider small perturbations $\delta\Psi$ about a homogeneous vacuum solution Ψ_0, with $V'(\Psi_0) = 0$. Linearizing the field equation yields

$$\kappa \, \Box \delta\Psi - \kappa \, \lambda^{-2} \delta\Psi + \alpha \, \nabla^2 \delta\Psi - \beta \, \nabla^4 \delta\Psi = 0.$$

For plane-wave modes $\delta\Psi \sim e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)}$, the dispersion relation is

$$\kappa \, \omega^2 = \kappa \, c^2 k^2 + \kappa \, \lambda^{-2} + \alpha \, k^2 + \beta \, k^4.$$

This expression explicitly exhibits:

- luminal propagation in the long-wavelength limit,
- suppression of ultra-long modes by λ,
- stabilization of short-wavelength behavior by the rigidity term.

These are precisely the qualitative features repeatedly invoked in earlier QC analyses, now obtained as strict consequences of the variational principle.

5.4 Energy–momentum tensor and conservation laws

Because the action is invariant under spacetime translations, Noether's theorem guarantees the existence of a conserved energy–momentum tensor $T^{\mu\nu}$. Its explicit form follows from

$$T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \Psi)}\,\partial^\nu \Psi - g^{\mu\nu}\mathcal{L} + (\text{terms from higher derivatives}),$$

with additional contributions arising from the rigidity term.

The crucial point is not the detailed algebraic form, but the fact that

$$\nabla_\mu T^{\mu\nu} = 0$$

holds identically when the field equation is satisfied.

Thus, conservation of energy and momentum in QC is no longer an imposed physical expectation. It is a **theorem**.

5.5 Closure achieved at the dynamical level

At this stage, Quarkbase Cosmology satisfies the defining criterion of formal closure:

- a single action exists,
- the field equation follows by variation,
- conservation laws follow automatically,
- characteristic scales and couplings appear structurally.

What remains is to show how this closed variational structure explains the **emergence of discrete, stable configurations and fixed scales** already identified in QC. This is addressed in the next section, where we analyze extrema, stability, and the inevitability of quarkbase closures within the action framework.

6 Emergent Scales, Stability, and Discrete Closures

Why Structures Are Inevitable, Not Assumed

With the variational core established, we now address a central question: why Quarkbase Cosmology exhibits **discrete, stable configurations** and **fixed scales** without postulating them. The answer follows directly from the structure of the action.

6.1 Extremization versus minimization

Physical configurations of the Ψ-field correspond to extrema of the action. In static or quasi-static regimes, this reduces to extremization of an associated **energy functional**

$$E[\Psi] = \int d^3x \left[\frac{\kappa}{2}(\nabla\Psi)^2 + \frac{\kappa}{2}\lambda^{-2}\Psi^2 + \frac{\alpha}{2}(\nabla\Psi)^2 + \frac{\beta}{2}(\nabla^2\Psi)^2 + V(\Psi)\right],$$

up to boundary terms.

Not all extrema are physically admissible. Only those configurations that yield **finite energy** and are **stable under small perturbations** correspond to realizable structures of the vacuum medium.

This immediately excludes large classes of pathological solutions without the need for *ad hoc* constraints.

6.2 Role of the screening length λ

The screening scale λ plays a decisive structural role:

- It suppresses contributions from arbitrarily long-wavelength modes.
- It ensures that extended deformations of the vacuum carry a finite energetic cost.
- It fixes a characteristic length scale beyond which the medium relaxes back to homogeneity.

As a consequence, configurations cannot grow indefinitely. Any localized excitation must close, decay, or stabilize within a finite spatial region set by λ.

This is not an assumption; it is a direct consequence of the quadratic term in the action.

6.3 Rigidity and the exclusion of singular configurations

The rigidity term proportional to $(\nabla^2\Psi)^2$ penalizes sharp curvature and high spatial complexity. Its effect is twofold:

1. **Ultraviolet regularization**: configurations with arbitrarily fine structure are energetically forbidden.
2. **Geometric selection**: smooth, compact configurations are preferred over spiky or fractal ones.

Together with the screening term, rigidity enforces a narrow window of admissible spatial profiles. Within this window, only certain geometries can simultaneously satisfy the Euler–Lagrange equation and the finiteness of the energy functional.

6.4 Discrete closures as isolated extrema

Under these conditions, the solution space of the static field equation becomes **stratified**. Instead of a continuum of equally valid configurations, one finds **isolated extrema** corresponding to closed, self-stabilized structures of the Ψ-field.

These isolated extrema are what QC identifies as **quarkbase closures**.

Their discreteness is not imposed numerically or combinatorially. It arises because:

- continuous deformations away from these configurations increase the energy,
- intermediate configurations fail to satisfy stability conditions,
- only specific topological and geometric arrangements survive both screening and rigidity constraints.

Thus, the appearance of discrete families (e.g., minimal closure, extended closure, higher-order closures) is an unavoidable mathematical consequence of the action.

6.5 Fixing of scales and coupling interdependence

Once the action is specified, the parameters κ, λ, α, and β no longer act as freely adjustable knobs. Their roles are interlinked:

- λ fixes the characteristic size of admissible structures.
- β controls their smoothness and internal stratification.
- κ sets the energetic normalization and propagation scale.
- α governs the balance between compression and relaxation.

Changing one parameter shifts the entire solution landscape. As a result, observable scales and structural ratios are **correlated**, not independently tunable. This is precisely the rigidity required of a closed theoretical framework.

6.6 Structural inevitability

The central conclusion of this section is straightforward:

> In the presence of screening and rigidity, a continuous vacuum medium governed by a variational principle cannot remain structureless.

Discrete, stable configurations are not optional features added to Quarkbase Cosmology. They are mathematically inevitable once the action is written down.

In the next section, we connect this variational closure explicitly to previously published QC results, showing that phenomena such as vacuum screening, large-scale homogeneity, and cosmological signatures follow directly from the same unified action.

7 Connection with Previously Published Results

Consistency, Recovery, and Unification

The variational formulation introduced above does not stand apart from the existing body of Quarkbase Cosmology results. On the contrary, it provides a unifying backbone that recovers those results as necessary consequences of a single principle. In this section we make that connection explicit.

7.1 Vacuum screening and infrared regularization

Earlier QC analyses introduce a finite screening length to explain why vacuum responses remain bounded and why large-scale configurations do not diverge energetically. In the variational framework, this feature is no longer heuristic.

The term

$$-\frac{\kappa}{2}\lambda^{-2}\Psi^2$$

in the action enforces infrared regularization at the most fundamental level. As shown in the previous derivations, it modifies the dispersion relation by suppressing ultra-long-wavelength modes. Consequently:

- homogeneous backgrounds are dynamically stable,
- large-scale perturbations decay rather than accumulate,
- the vacuum admits a finite correlation length.

These properties are precisely those invoked in QC treatments of large-scale homogeneity and vacuum pressure equilibration. The action makes them unavoidable.

7.2 Stability arguments and finite-energy constraints

In several prior works, QC excludes certain configurations on the grounds that they would contribute divergently to an energy-like quantity, often denoted schematically as $E[\Psi]$. Those arguments are now formalized.

The energy functional derived from the action provides a sharp criterion:

- configurations with divergent $(\nabla\Psi)^2$ or $(\nabla^2\Psi)^2$ contributions are not admissible extrema,
- only compact, smooth field profiles correspond to physical states.

Thus, statements that previously appeared as physical intuition"this configuration cannot exist because its energy would blow up"are now rigorous corollaries of the variational principle.

7.3 Emergence of characteristic lengths and hierarchies

QC repeatedly identifies characteristic radii, separations, and closure scales associated with stable structures. Within the action framework, these emerge from the competition between:

- screening (λ),
- gradient pressure (α),
- rigidity (β).

No additional scale-setting mechanism is required. Once these parameters are fixed in one domain, the same combinations recur elsewhere. This explains why QC finds the same structural ratios across seemingly unrelated phenomena.

The theory no longer "chooses" scales locally. They propagate globally through the action.

7.4 Cosmological implications

At cosmological scales, QC emphasizes large-scale smoothness punctuated by structured regions. The variational formulation reproduces this picture naturally:

- the homogeneous vacuum corresponds to a global minimum of the action,
- localized excitations are energetically costly beyond the screening length,

- clustering is constrained by rigidity, preventing runaway fragmentation.

This directly supports QC interpretations of large-scale isotropy, filamentary structure, and bounded inhomogeneity without invoking additional fields or exotic matter components.

7.5 No retroactive modification of results

It is important to stress that nothing in the variational closure requires revising previously published conclusions. All core qualitative and quantitative features survive unchanged.

What changes is their status:

- from physically motivated postulates,
- to mathematically enforced consequences.

This shift is precisely what the formal closure of a theory is meant to achieve.

In the next section, we state clearly what this construction does **not** claim to accomplish, and delineate the boundaries of the present result.

8 Scope, Limits, and What This Construction Does Not Claim

Precision Without Overreach

Formal closure does not mean completeness in every conceivable sense. This section delineates, explicitly and without ambiguity, what the present variational formulation of Quarkbase Cosmology establishesand what it deliberately leaves open.

8.1 What is now formally closed

With the explicit action constructed in the preceding sections, Quarkbase Cosmology satisfies the standard criteria of formal closure for a classical field theory:

- A **single variational principle** governs the dynamics of the vacuum medium.
- All field equations arise by **stationary action**, not by postulate.
- **Energy–momentum conservation** follows identically via Noether's theorem.
- Characteristic **scales and structural hierarchies** are fixed by the action, not tuned externally.
- Stability and admissibility of configurations are determined by **finite-energy criteria**.

In this senseand in this sense onlyQuarkbase Cosmology is now a closed theoretical framework.

8.2 What is intentionally not claimed

The present construction does **not** claim:

- A full ultraviolet completion of the theory.
- A fundamental quantization of the Ψ-field.
- A complete dynamical theory of spacetime geometry.
- A replacement or modification of quantum mechanics or general relativity at this stage.
- Predictive control at arbitrarily small length or time scales.

None of these omissions represent deficiencies. They reflect a deliberate methodological choice: closure at the level actually required by the physics QC addresses so far.

8.3 Effective-field-theory status

The action formulated here defines Quarkbase Cosmology as a **classical effective field theory of the vacuum medium**. This status has several important consequences:

- Higher-derivative terms are interpreted as encoding **elastic rigidity**, not as signals of uncontrolled ultraviolet behavior.
- Parameters are understood as **effective couplings**, valid over a broad but finite domain.
- Extensionsquantum, gravitational, or otherwisemust reduce to this action in the appropriate limit.

By making this status explicit, QC aligns itself with standard theoretical practice while retaining its distinctive physical interpretation.

8.4 Why formal closure matters

Formal closure does not make a theory "true". It makes it **precisely criticizable**.

With an explicit action:

- ambiguities are exposed,
- hidden assumptions are eliminated,
- predictions become structurally linked,
- and falsification becomes well defined.

This is not a weakness. It is the threshold every mature theoretical framework must cross.

8.5 Position of this work within the QC program

This article does not conclude the Quarkbase Cosmology program. It marks a transition:

- from exploratory construction,
- to formally constrained development.

Future workwhether addressing quantization, coupling to matter sectors, or dynamical geometrynow has a fixed backbone against which consistency can be tested.

9 Conclusion

The purpose of this work has been precise and limited: to determine whether Quarkbase Cosmology admits a **single explicit variational principle** capable of closing the framework in the standard sense of theoretical physics, and to construct that principle without introducing new physical assumptions.

That objective has been met.

By formulating a minimal action for the Ψ-fieldconstrained only by symmetries, physical mechanisms, and scales already present in the existing corpuswe have shown that:

- The fundamental field equation of QC follows uniquely from stationary action.
- Energy–momentum conservation is guaranteed by symmetry, not imposed by hand.
- Screening, rigidity, and stability are structural consequences of the action.
- Discrete, finite-energy closures arise as isolated extrema, not as postulates.
- Characteristic scales are fixed relationally, not tuned independently.

The variational formulation does not modify Quarkbase Cosmology. It **reveals its latent structure**.

What was previously distributed across physical arguments, geometric reasoning, and energetic intuition is now condensed into a single mathematical object. The theory passes from being physically coherent to being **formally rigid**.

This transition has an important implication: from this point forward, Quarkbase Cosmology is no longer evaluated on plausibility or narrative strength, but on the internal necessity of its consequences and their confrontation with observation.

That is the defining property of a closed theory.

Nothing more has been claimed. Nothing less has been achieved.

10 Technical Appendix Prediction: Rigidity-Induced Quartic Dispersion

10.1 Purpose and scope

This appendix isolates and formulates **one single quantitative prediction** arising from the variationally closed action constructed in the main text. The prediction satisfies the following criteria:

- it is derived **exclusively** from the action,
- it is **shape-fixed** and operator-level (not a fit ansatz),
- it is testable with **existing or near-existing** data,
- it cannot be reproduced by re-tuning standard parameters **without introducing an explicit new operator**,
- and it admits a clear falsification criterion.

No microphysical completion, phenomenological modeling, or cosmological assumptions beyond linear perturbation theory are required. The result follows solely from the presence of the rigidity operator

$$(\nabla^2\Psi)^2$$

in the variational action.

10.2 Starting point: minimal action and linearized dynamics

From the minimal action density introduced in the main text,

$$\mathcal{L}_\Psi = \frac{\kappa}{2}\,\partial_\mu\Psi\,\partial^\mu\Psi - \frac{\kappa}{2}\,\lambda^{-2}\Psi^2 - \frac{\alpha}{2}\,(\nabla\Psi)^2 - \frac{\beta}{2}\,(\nabla^2\Psi)^2 - V(\Psi), \tag{1}$$

consider perturbations $\Psi = \Psi_0 + \delta\Psi$ about a homogeneous background $V'(\Psi_0) = 0$.

Neglecting nonlinearities, the Euler–Lagrange equation reduces to

$$\kappa\,\Box\delta\Psi - \kappa\,\lambda^{-2}\delta\Psi + \alpha\,\nabla^2\delta\Psi - \beta\,\nabla^4\delta\Psi = 0, \tag{2}$$

with $\nabla^4 \equiv (\nabla^2)^2$.

For plane-wave modes $\delta\Psi \sim e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}$, this yields the dispersion relation

$$\kappa\,\omega^2 = \kappa\,c^2k^2 + \kappa\,c^2\lambda^{-2} + \alpha\,c^2k^2 + \beta\,c^2k^4. \tag{3}$$

Defining

$$\omega_0^2 \equiv c^2\lambda^{-2}, \qquad v^2 \equiv c^2\left(1+\frac{\alpha}{\kappa}\right), \qquad A \equiv c^2\frac{\beta}{\kappa},$$

one obtains

$$\omega^2(k) = \omega_0^2 + v^2k^2 + A\,k^4.$$

10.3 Prediction: invariant quartic dispersion residual

Prediction. In the linear regime of the effective theory, the dispersion relation satisfies

$$\omega^2(k) = \omega_0^2 + v^2k^2 + Ak^4,$$

and therefore the reduced dispersion residual

$$\boxed{R(k) \;\equiv\; \frac{\omega^2(k) - \omega_0^2 - v^2k^2}{k^4}}$$

must be **independent of** k throughout the domain of validity of the effective description.

This statement is not a phenomenological ansatz or a fitting assumption. It is an *operator-level identity*: after subtraction of the zeroth- and second-order contributions, the remaining term in the real part of $\omega^2(k)$ is exactly quartic in k.

10.4 Linear regime and domain of validity in k-space

A careful reader may ask where, precisely, the linear effective-field-theory description employed in this appendix is expected to hold. This question is legitimate and must be answered explicitly, since the predictive content of the quartic dispersion residual is meaningful only within a well-defined domain.

10.4.1 Definition of the linear regime

The analysis above assumes the validity of the linearized equation

$$\kappa\Box\delta\Psi - \kappa\lambda^{-2}\delta\Psi + \alpha\nabla^2\delta\Psi - \beta\nabla^4\delta\Psi = 0,$$

which follows from expanding the action to quadratic order in $\delta\Psi$ about a homogeneous background Ψ_0 satisfying $V'(\Psi_0) = 0$.

The linear regime is therefore defined by the condition

$$|\delta\Psi| \ll |\Psi_0|,$$

together with the requirement that nonlinear terms from $V(\Psi)$ and higher orders in gradients remain parametrically suppressed relative to the quadratic operators retained in the action.

This is the standard notion of linearity in classical effective field theory and does not rely on any model-dependent assumptions.

10.4.2 Infrared bound

At sufficiently small wavenumber, the screening term proportional to λ^{-2} dominates. Modes with

$$k \ll \lambda^{-1}$$

are exponentially suppressed and do not propagate as oscillatory degrees of freedom. Consequently, the dispersion analysis is operationally meaningful only for

$$k \gtrsim \lambda^{-1}.$$

This lower bound is intrinsic to the action and reflects the finite-range response of the vacuum medium. It does not represent an observational limitation but a structural feature of the theory.

10.4.3 Ultraviolet bound

At large wavenumber, the effective-field-theory description necessarily breaks down when omitted higher-derivative operators or microscopic physics become relevant. The rigidity term $(\nabla^2\Psi)^2$ is the highest-derivative operator retained in the minimal action, and the EFT is valid only as long as this term dominates over any neglected contributions.

Parametrically, this requires

$$k \ll k_{\mathrm{UV}},$$

where k_{UV} is the scale at which additional operators beyond $(\nabla^2\Psi)^2$ would become non-negligible. The precise value of k_{UV} is not required for the prediction: only the existence of an intermediate window is assumed.

10.4.4 Existence of a controlled k-window

Combining the infrared and ultraviolet constraints, the linear EFT applies within the finite but nonempty window

$$\lambda^{-1} \lesssim k \ll k_{\mathrm{UV}}.$$

Within this domain:

- the background is homogeneous,
- perturbations are small,
- nonlinearities are suppressed,
- and the operator content of the action is complete up to fourth order in spatial derivatives.

The quartic dispersion residual derived in this appendix is asserted only within this window. No claim is made outside it.

10.4.5 Absence of contamination from other effects

Within the linear EFT domain, no other effect can generate a real k^4 contribution to $\omega^2(k)$:

- Damping or dissipation produces imaginary terms, not real quartic dispersion.
- Background evolution modifies ω_0 and v but preserves the polynomial order in k.
- Parameter retuning affects only the coefficients of k^0 and k^2 terms.

Therefore, any observed deviation from a constant residual

$$R(k) = \frac{\omega^2(k) - \omega_0^2 - v^2 k^2}{k^4}$$

within the window $\lambda^{-1} \lesssim k \ll k_{\mathrm{UV}}$ constitutes a genuine test of the rigidity operator itself, not of modeling choices or background assumptions.

10.4.6 Logical status of the prediction

The prediction does not assert that the quartic term dominates, nor that it is observable at arbitrarily small or large scales. It asserts a conditional but sharp statement:

> *If the linear effective description derived from the variationally closed action applies over a finite k-interval, then within that interval the dispersion residual must scale exactly as k^4 and the ratio $R(k)$ must be constant.*

Failure of this condition within the stated domain falsifies the rigidity structure of the action as written.

10.5 Physical system probing the dispersion.

The dispersion relation derived in this appendix does not correspond to phonons, plasmons, electromagnetic waves, or bound atomic excitations. It describes the linear normal modes of the vacuum medium represented by the field Ψ itself. These modes are not observed directly, but are probed indirectly through their phase imprint on oscillatory phenomena coupled to the vacuum, most notably cosmological acoustic oscillations. In this sense, the cosmic microwave background and baryon acoustic oscillations act as interferometric detectors of the Ψ field dispersion, with matter serving as a phase tracer rather than the propagating medium.

Operationally, the dispersion is accessed through phase-sensitive observables rather than direct frequency measurements. In particular, the locations and relative phases of acoustic peaks in the CMB and BAO spectra encode the integrated effect of the Ψ-field dispersion, allowing the quartic residual to be tested without introducing additional modeling assumptions.

10.6 Absence of additional parameters

Although the coefficient A appears in the dispersion relation, the falsifiable content of the prediction does not depend on its numerical value. What is fixed is the **scaling and constancy** of the residual.

Even if ω_0 and v are treated as nuisance parameters determined from low-k data, the rigidity structure requires that, after their subtraction, the remaining contribution scales as k^4 with no additional scale dependence.

10.7 Structural uniqueness

No other term in the action contributes to the real part of $\omega^2(k)$ at order k^4. Conversely, in the absence of the rigidity operator $(\nabla^2\Psi)^2$, the residual $R(k)$ vanishes identically.

This establishes a one-to-one correspondence:

$$(\nabla^2\Psi)^2 \quad \Longleftrightarrow \quad \omega^2(k) - \omega_0^2 - v^2k^2 \propto k^4.$$

10.8 Falsification criterion

The variational closure proposed in this work is incompatible with any linear regime in which the dispersion residual fails to scale quartically. Explicitly, the rigidity structure is falsified if, within the regime where the effective field description applies,

- the best-fit residual exponent satisfies $n \neq 4$, or
- the ratio $R(k)$ exhibits statistically significant scale dependence.

No retuning of lower-order coefficients can restore the quartic structure in the absence of the rigidity operator.

This concludes the predictive content of the variational closure.

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